Unit 6

Similarity

Unit Goals –

1. Solve proportions and simplify ratios.

2. Apply ratios and proportions to solve word problems.

3. Recognize, determine, and apply scale factors.

4. Identify similar figures.

5. Apply properties of similar figures to find missing angle measures and side lengths of figures.

Ratio – <u>the relation of two numbers through division.</u>

We have seen examples of ratios before...

$$\frac{3}{4} \qquad \frac{5}{2} \qquad$$ ...fractions are ratios.

Ratios can be written in three different ways:

1.) $\dfrac{3}{4}$ Fraction

2.) 3:4   Colon

3.) 3 to 4   "to"

These are all equivalent ratios and they represent the ratio of the number 3 to the number 4.

Ratios are usually expressed in simplest form. This means that for a final answer you should reduce all ratios. You reduce ratios just like you would reduce a fraction.

Ex 1.) $6:18$

$\boxed{1:3}$

$$\frac{6}{18} = \frac{1}{3}$$

Ex 2.

$$\frac{12}{8} = \frac{3}{2}$$

Do not change improper fractions into mixed numbers.

Ex 3.

$$\frac{16}{4} = \frac{4}{1}$$

Leave whole numbers in fraction form.

When simplifying ratios, the units of each number must be the same.

Ex 1. $\dfrac{5mm}{3cm}$ $\qquad$ $\dfrac{5mm}{30mm}$ $\qquad$ $\dfrac{1mm}{6mm}$

Ex 2.

$\dfrac{4yd}{14ft}$ $\qquad$ $\dfrac{12ft}{14ft}$ $\qquad$ $\dfrac{6ft}{7ft}$

**You'll usually convert to the smaller measurement

Steps for completing ratio word problems:

1. Create an equation using the ratio.

2. Solve the equation for x.

3. Use x to determine each value.

4. Check to make sure the sum and the ratio of the values is correct.

How can we apply ratios to word problems?

Example 1 – The ratio of two supplementary angles is 4:11. Find the measure of each angle.

$$4x + 11x = 180$$

$$15x = 180$$

$$x = 12$$

$$4x = 4(12) = \boxed{48}$$

$$11x = 11(12) = \boxed{132}$$

Check: 48 + 132 = 180

and

$$\frac{48}{132} = \frac{4}{11}$$

Example 2 – The measure of the angles in a triangle are in the ratio 3:4:5. Find the measure of each angle.

$3x + 4x + 5x = 180$

$12x = 180$

$x = 15$

$3x = 3(15) = \boxed{45}$

$4x = 4(15) = \boxed{60}$

$5x = 5(15) = \boxed{75}$

Check: $45 + 60 + 75 = 180$

and

$45:60:75 = 3:4:5$

Example 3 – <u>$1000</u> prize money is to be allotted to the first, second, and third place winners of a contest in the ratio of <u>4:3:1</u>. Determine how much money each winner will receive.

$$4x + 3x + 1x = 1000$$
$$8x = 1000$$
$$x = 125$$

$$4x = 4(125) = \boxed{\$500}$$
$$3x = 3(125) = \boxed{\$375}$$
$$1x = 1(125) = \boxed{\$125}$$

<u>Check</u>: 500 + 375 + 125 = 1000

and

500:375:125 = 4:3:1

Proportion – <u>a proportion is an equation stating that two ratios are equal.</u>

Extremes: <u>first and last terms</u>

Means: <u>middle terms</u>

$$\frac{a}{b} = \frac{c}{d}$$

$$a : d = b : c$$

Properties of Proportions

$$\frac{a}{b} = \frac{c}{d}$$ is equivalent to.....

a.) $ad = bc$

c.) $\frac{b}{a} = \frac{d}{c}$

b.) $\frac{a}{c} = \frac{b}{d}$

d.) $\frac{a+b}{b} = \frac{c+d}{d}$

We can use algebra to solve for unknowns within proportions. To solve a proportion you must:

1. Cross Multiply

2. Write an equation

3. Solve the equation for the variable

Ex 1.

$$\frac{4}{5} = \frac{x}{10}$$ $(4)(10) = (5)(x)$ $40 = 5x$

$$\boxed{x = 8}$$

$$\frac{4}{5} = \frac{8}{10}$$

Ex 2.

$$\frac{2x+1}{4x-1} = \frac{2}{3}$$

$$3(2x+1) = 2(4x-1)$$

$$6x+3 = 8x-2$$

$$5 = 2x$$

$$\boxed{x = 2.5}$$

Where else do we see ratios and proportions?

On a map or blueprints there is a <u>scale factor</u>. The scale factor tells you the dimensions and measurement of objects that are represented by smaller versions in maps, blueprints, scale drawings, and scale models.

Example: On a road map, the scale factor may read 1 inch to 100 miles. That means that every 1 inch on the map represents 100 miles in reality.

Ex 1. On a map every <u>1 inch</u> equals <u>50 miles</u>. You measure the distance between your house and your friend's house as <u>3.5 inches</u> on the map. How many miles apart do you really live?

$$\frac{1in}{50mi} = \frac{3.5in}{x}$$

*Always keep consistent units.

$$x = (50)(3.5) = 175mi$$

Answer: 175 miles

Ex 2. On a scale drawing every 2 cm represents 5 m. If you measure an object on the drawing as 3.7cm, how long is it really?

$$\frac{2cm}{5m} = \frac{3.7cm}{x}$$

$$2x = (5)(3.7)$$

$$2x = 18.5$$

$$x = 9.25m$$

Answer: 9.25 m

Ex 3. If <u>3 bags</u> of apples cost <u>$16</u>, then how much does <u>12 bags</u> of apples cost?

Decide on the ratio: $\dfrac{\underline{Bags\ of\ Apples}}{Cost}$

$$\frac{3}{16} = \frac{12}{x}$$

$$192 = 3x$$

$$64 = x$$

Answer: $64